NASA Contractor Report 181618

ICASE REPORT NO. 88-9

ICASE

AN APPLICATION OF EIGENSPACE METHODS

TO SYMMETRIC FLUTTER SUPPRESSION

AD-A192 758

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Contract No. NAS1-18107
January 1988

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING
NASA Langley Research Center, Hampton, Virginia 23665

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AN APPLICATION OF EIGENSPACE METHODS
TO SYMMETRIC FLUTTER SUPPRESSION

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ABSTRACT.

An eigenspace assignment approach to the design of parameter insensitive control laws for linear multivariable systems is presented. The control design scheme utilizes flexibility in eigenvector assignments to reduce control system sensitivity to changes in system parameters. The methods involve use of the singular value decomposition to provide an exact description of allowable eigenvectors in terms of a minimum number of design parameters. In a design example, the methods are applied to the problem of symmetric flutter suppression in an aeroelastic vehicle. In this example the flutter mode is sensitive to changes in dynamic pressure and eigenspace methods are used to enhance the performance of a stabilizing minimum energy/ linear quadratic regulator controller and associated observer. Results indicate that the methods provide feedback control laws that make stability of the nominal closed loop systems insensitive to changes in dynamic pressure.

i

AN APPLICATION OF EIGENSPACE METHODS
TO SYMMETRIC FLUTTER SUPPRESSION

1. INTRODUCTION.

A common problem in flight control applications is to design feedback and observer gain matrices so that the system remains stable and satisifies given performance requirements over a range of flight conditions. A basic approach is to fix a flight condition as a design point and to synthesize gain matrices so that stability and performance requirements are met at the design point.Nearby flight conditions yield evalution models. If performance requirements are not met at the evaluation points, one returns to the design model and adjusts the existing design to meet more stringent requirements. In such cases it frequently occurs that certain variables used for performance evaluation are more sensitive to changes in the design point than others and a reduction in sensitivity can be explicitly considered in the design process. In this paper eigenspace methods are presented for the design of parameter insensitive control laws. The methods are applied to the problem of symmetric flutter suppression in an aeroelastic vehicle.

The ability to shape the fundamental modes of a system by modifying selected eigenvalues and eigenvectors underlies the appeal of eigenspace methods for control system design. The basic freedoms and limitations of eigenstructure assigment methods by full state feedback or output feedback are presented in the papers by Moore [12], by Srinathkumar [14], by Andry, Shapiro and Chung [2], and by Kautsky, Nichols and Van Dooren [7]. The utility of eigenstructure assignment methods in observer designs is discussed in the work of Kazerooni and Houpt [8]. In these methods feedback and observer gain matrices are determined to yield a desired eigenstructure. Stability and transmission considerations motivate the location of desired eigenvalues,whereas the ability to shape the system response [2, 12], to enhance system performance [5], to reduce system sensitivity [15], or to design robust control laws [7, 8] motivates the selection of desired eigenvectors.

In this paper, vector space methods are presented for eigenspace assignment which take into consideration eigenvalue sensitivity to plant parameter variations and performance constraints. The procedure is formulated as a constrained optimization problem in Section 2 along with a description of eigenspace assignment methods for the determination of feedback and observer gain matrices. Formulation of the procedure as a constrained optimization problem allows explicit consideration of costs associatied with control effort in the design procedure. Eigenvalue sensitivity can be expressed explicitly in terms of closed loop eigenvectors and, consequently, the freedom available in assigning eigenvectors can be directly related to sensitivity reduction. In Section 2, the singular value decomposition [9] is used to provide a basis for the attainable eigenvectors associated with a desired eigenvalue. Thus an explicit parameterization/coordinatization of the attainable eigenvectors is obtained. An alternate algorithm for computing a basis for the attainable eigenvectors was presented in the work of Porter and D'Azzo [13] and, more recently, by Kautsky, Nichols and Van Dooren

[7]. Some eigenvector assignment procedures require the control designer to specify a desired eigenvector and the nearest attainable eigenvector is then computed [2], [5]. The approach advocated, here, involves a precise description of the attainable eigenvectors and results in the display of the design freedom in terms of a minimum number of independent parameters.

A flutter suppresssion/gust load alleviation problem is formulated in Section 3 and the results of an extensive design example are presented in Section 4. In this example, the flutter mode is sensitive to changes in dynmaic pressure and eigenspace methods are used to enhance the performance properties of a LQR/LTR designed compensator. Results indicate that the methods provide feedback control laws which make the stabllity of the nominal closed loop system insensitive to changes in dynamic pressure.

2. PARAMETER INSENSITIVE CONTROL SYSTEM DESIGN.

In this section a procedure based upon eigenstructure assignment methods for the design of parameter insensitive control laws is presented. A review of eigenstructure assignment methods useful in the design of full state feedback control laws and state observers is included. In this paper the singular value decomposition is used to provide a coordinatization of the allowable eigenvectors, which arise in full state feedback designs. These coordinates become the design parameters for sensitivity reduction.

Eigenstructure assignment. Consider the linear system

$$dx/dt = A x + B u + \Gamma \eta \tag{1}$$

where A, B, and Γ denote matrices of appropriate dimensions and x, u, η denote state, control, and disturbance variables respectively. Assuming full state feedback control, i. e. $u = -Kx$, the eigenvalues of A - BK can be arbitrarily assigned through a proper choice of the gain matrix K if and only if the system (1) is completely controllable. If (1) is uncontrollable then uncontrollable eigenvalues cannot be altered by a state variable feedback control law $u = -Kx$.

The ability to assign eigenvalues and eigenvectors through state variable feedback control laws is summarized as follows [3, 7, 12, 14]. Let $\Lambda = \{ \lambda_i : i=1, ..., n \}$ be a self conjugate set of distinct complex numbers. Here Λ denotes a set of desired closed loop eigenvalues and must include all uncontrollable eigenvalues of the system. There exists a feedback gain matrix K such that $(A - BK) v_i = \lambda_i v_i$ for $i = 1, ..., n$ if and only if

i) $\{ v_i \}_{i=1,n}$ are linearly independent in C^n and $v_i = \bar{v}_j$ if $\lambda_i = \bar{\lambda}_j$

ii) $\begin{bmatrix} v_i \\ w_i \end{bmatrix}$ belongs to the Ker $[\lambda_i I - A, B]$ for $i = 1, ..., n$

iii) $w_i = K v_i$ for $i = 1, ..., n$

In this case the feedback gain matrix K must satisfy K = [w_1, ... w_n]$^{-1}$ [v_1, ... v_n].
Thus λ is an eigenvalue for A - BK with corresponding eigenvector v provided

$$[\lambda I - A, B] \begin{bmatrix} v \\ Kv \end{bmatrix} = 0 \tag{2}$$

If W denotes a matrix whose columns form a basis for the Ker [$\lambda I - A$, B] then

$$\begin{bmatrix} v \\ Kv \end{bmatrix} = Wc = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} c \tag{3}$$

where c denotes a vector whose components are the coordinates of [v , (Kv)^T]^T with respect to the columns of W. Thus (3) provides a parameterization or coordinatization of the allowable eigenvectors. If λ is a controllable eigenvalue it may be shown that the dimension of Ker [$\lambda I - A$, B] equals the number of columns of B and the columns of W_1, see (3) above, are independent if B is of full rank [12]. The singular value decomposition [7, 9], SVD, provides an efficient numerical method to compute an orthonormal basis for Ker [$\lambda I - A$, B]. If the SVD of [$\lambda I - A$, B] is

$$[\lambda I - A, B] = [U_1, U_2] \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} [V_1, V_2]^T \tag{4}$$

then the columns of W = V_2 form an orthonormal basis for the Ker [$\lambda I - A$, B].

Equation (3) provides a parameterization or coordinatization of the attainable eigenvectors associated with λ and, consequently, completely describes the freedom in assigning a particular eigenvector. One should note that the number of free parameters describing an atttainable eigenvector equals the rank(W_1) $\leq$ dim Ker [$\lambda I - A$, B]. These free parameters will be utilized as control design parameters in the application to follow. Further characterizations of the attainable eigenvectors and the number of degrees of freedom in eigenvector assignment are presented in the work of Kautsky, Nichols and Van Dooren [7].

In this design example the coordinates of an attainable eigenvector are modified to obtain feedback control laws which are insensitive to changes in a model parameter. The approach requires that derivatives of eigenvalues with respect to system parameters be computed. It is well known [7, 10] that if λ is a distinct eigenvalue of a matrix Q(q), depending on a parameter q, and u', v are left and right eigenvectors corresponding to λ , respectively, then

$$\frac{d\lambda}{dq} = \frac{u' \, dQ/dq \, v}{u' \, v} \tag{5}$$

The dependence of this sensitivity on the eigenvectors u' and v indicates a potential reduction in sensitivity by a suitable choice of eigenvectors. Bounds on eigenvalue sensitivities are directly

related to the condition number of the modal matrix for the closed loop systems and are discussed in [7].

Eigenstructure assignment in observer designs. In a recent paper [8], Kazerooni and Houpt presented a procedure for loop transfer recovery based upon eigenstructure assignment of observers. This procedure for observer design will be used in the example to follow. These same methods have been previously applied in the design of an active flutter suppression system by Garrard, Liebst, and Farm [5]. For completeness the procedure is reviewed in this section.

Suppose now only output feedback is available and an observer is to be employed to estimate the state, then equation (1) is coupled with the following output, observer, and feedback equations:

$$y = C x \tag{6}$$

$$dz/dt = A z + B u + H (y - C z) \tag{7}$$

$$u = -K z \tag{8}$$

Here K and H denote feedback and observer gain matrices, respectively.

Setting $u = -K z + u_o$ in place of (8) and $e = x - z$, one obtains

$$de/dt = (A - HC) e + B u_o + \Gamma \eta . \tag{9}$$

In terms of transfer functions $T_{zu}(s) = T_{xu}(s) - T_{eu}(s)$. For recovery it is desireable to have $T_{zu}(s) = T_{xu}(s)$ or $T_{eu}(s) = 0$.

The condition $T_{eu}(s) = 0$ can be described in terms of transmission properties of (1) and (7). A complex number ξ is called an invariant zero [11] of the system (1) and (6) with left zero direction $[v' , \mu']$ provided

$$[v', \mu'] \begin{bmatrix} \xi I - A & B \\ C & 0 \end{bmatrix} = 0 \tag{10}$$

and $[v' , \mu'] \neq 0$. If a gain matrix H can be chosen which assigns the eigenvalues $\{\lambda_i : i=1, ...,n\}$ of A - HC to invariant zeros $\{ \xi_i : i=1, ..., n\}$ of (1) and (6) and left eigenvector v_i' to corresponding left zero direction v_i' then, necessarily,

$$[v_i' , v_i' H] \begin{bmatrix} \lambda_i I - A & B \\ C & 0 \end{bmatrix} = 0$$

Moreover, since the zero state response of (9) is

$$e(t) = \sum_{j=1}^{n} u_j v_j' B e^{\lambda_j t} * u_0(t)$$

where u_j v_j denote right and left eigenvectors of A corresponding to λ_j, it follows that e(t) = 0, i.e.

$T_{eu} = 0$. Thus for recovery it is desirable to place the eigenvalues of A - HC at the invariant zeros of (1) and (6) and the left eigenvectors at corresponding left zero directions.

Numerical solution of (10) reduces to the solution of a generalized eigenvalue problem and existing computer software [5, 6, 7] can be used to obtain the invariant zeros and left zero directions. In this procedure the desired eigenstructure represents an ideal. There is no guarantee that the desired eigenstructure is attainable. In the design example to follow nearest attainable eigenvectors are computed using the methods presented in [6]. Once desired eigenvalues and attainable eigenvectors have been specified, the observer gain matrix can be computed using eigenstructure assignment methods. For further discussion on the application of this procedure see [3] and [6]. If the observer gain H(p) depends on a parameter p and H(p)/p $\rightarrow$ BW as p $\rightarrow \infty$ where W is a nonsingular matrix then a standard arguement [10] implies that the eigenvalues of A - HC converge to the transmission zeros of (1) and (6). The control transfer function for the system described by equations (1), (6), (7), and (8) is u = - K(s)y with

$$K(s) = K (sI - A + BK + HC)^{-1} H.$$

Let G(s) denote the transfer function associated with (1) and (7), i.e.

$$G(s) = C (sI - A)^{-1}B.$$

In this case Doyle and Stein [6] have also shown that K(s)G(s) $\rightarrow$ KG(s) pointwise. Thus, from another viewpoint, it is desireable to place the eigenvalues of the observer at the transmission zeros of the system (1) and (6).

Parameter insensitive control design. Here the objective is to determine the design parameters which minimize the sensitivity of certain performance variables while, at the same time, maintaining other performance variables within prescribed bounds. By introducing an appropriate penalty function, this problem can be formulated as a constrained optimization problem.

In this paper, the design variables are those eigenvalues and eigenvectors of the system matricies $A(q_o)$ - $B(q_o)K$ and $A(q_o)$ - $HC(q_o)$ which may be modified in order to reduce sensitivity. The design parameters are the real and imaginary parts of the designated eigenvalues and the coordinates of the associated eigenvectors. In the design process employed herein, a basis for the allowable eigenvectors must be computed for each eigenvector to be modified, see equation (3). The coordinates of an eigenvector with respect to this basis become the design parameters that relate to eigenvector selection. If a desired eigenvalue is to be left unaltered throughout a design, that is eigensystem design freedoms are not fully utilized, then this basis need only be computed once.

Let α denote the vector of design parameters. In the design procedure certain performance variables, those which measure sensitivity to plant variations, are to be minimized. Let s(α) denote a vector of such variables. In the design example presented in Section 4, s(α) will denote eigenvalue sensitivity to plant parameter variation. Let p(α) denote a vector of performance variables to be kept within prescribed bounds. The prescribed lower and upper bounds for the i [th]

$$LB_i < p_i(\omega)_j \le UB_i, \ldots$$

$$\ldots \{ UB_i, \ UB - p(\omega)_j\}$$

... function

... performance constraint

... in order to obtain an

... DESCRIPTION

... tion and gust load alleviation to ... vehicle wing with three control surfaces and three vertical ... Figure 3.1. State space equations for control design were obtained by

§ sensor #1

▲ sensor #2

■ sensor #3

Figure 3.1 Hypothetical Wing

employing a two-and [...]

design and evaluation [...]

$$dx/dt = Ax$$

$$y = Cx$$

where A is $n \times n$, B is n [...]

associated with general [...]

surface positions [...]

control inputs and is x [...]

are associated with vertical [...]

depicted in Figure 3.2. The [...]

depicted in Figure 3.3. That [...] the damping and [...] deflection [...]

The point chosen for control [...] $q_0 = 468$ lb [...] that point serves as evaluation point

The control objective is to design feedback and observer gain matrices which stabilize that system

over a range of values of dynamic pressure and which keep control surface deflections and rates

within prescribed bounds. The maximum deflections and rates are set at 15° and 740°/sec,

respectively. The control law is to be designed so that saturation of controls does not occur for a

input gust spectrum having an rms intensity of 12 ft/sec.

$$
\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{bmatrix} =
\begin{bmatrix}
0 & A_{12} & 0 & 0 & 0 \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\
0 & A_{32} & A_{33} & A_{34} & A_{35} \\
0 & 0 & 0 & A_{44} & 0 \\
0 & 0 & 0 & 0 & A_{55}
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} +
\begin{bmatrix} 0 \\ 0 \\ 0 \\ B_4 \\ 0 \end{bmatrix} u +
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ B_5 \end{bmatrix}
$$

Figure 3.2. Control system structure.

3. DESIGN EXAMPLE.

At the design point q_0 [...] assemble a stabilizing observer-based design. The state feedback

control law $u = K(q_0) x$ was [...] and [...] are selected in the [...]. The

weighting matrices for control [...]

respectively, see Table 3.1 [...]

unstable open loop pole [...]

remain unaltered.
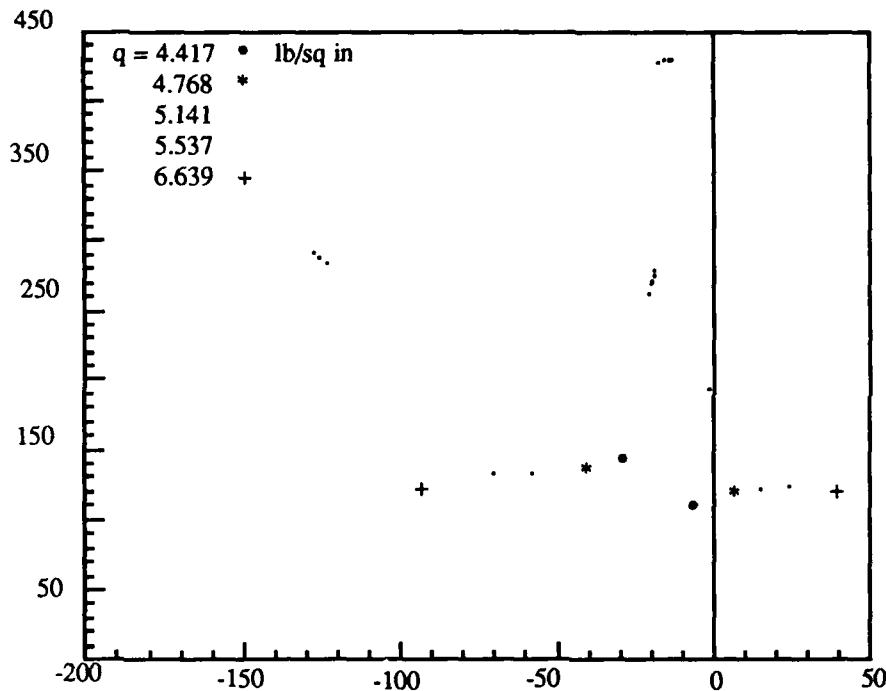
Figure 3.3 Locus of Open Loop Poles with Dynamic Pressure.

At the design point the controlled system

$$dx/dt = (A(q_o) - B(q_o)K(q_o)) x + \Gamma(q_o) \eta$$

exhibits good performance and disturbance rejection properties. The rms values listed in column LQR of Table 4.1 indicate acceptable control surface activity and loads for a 12 ft./sec. rms gust velocity. In addition, the minimum singular value for the return difference matrix $I + K(q_o)$ $(sI-A(q_o))^{-1}B(q_o)$ over the frequency range $10 \leq \omega \leq 300$ is approximately one, indicating good disturbance rejection properties.

At this design point an observer was determined by attempting to place the eigenvalues of A - HC at the transmission zeros of the plant, $C(q_o)(sI - A(q_o))^{-1}B(q_o)$, and corresponding eigenvectors at the left zero directions. The open loop eigenvalues, invariant zeros and actual assignments are listed in Table 4.2. Here eigenvalues 1-10 correspond to position and rate variables, 11-15 correspond to unsteady aerodynamic states, 16-24 correspond to actuator states, and 25-26 correspond to gust states. The zeros fall in three categories: those with large magnitude, those of the same order of magnitude and those with small magnitude relative to the magnitude of the open loop eigenvalues. The six small magnitude zeros result from computattional errors and should be set to zero. Actual pole-zero assignments are listed in columns I and II of Table 4.2. The desired eigenvalues and eigenvectors for the assignment listed in column I of Table 4.2 were made as follows: zeros of the same order of magnitude as those of the open loop poles (and corresponding left zero direction) were assigned to the nearest open loop eigenvalue (and

	LQR	LQR/LTR
Control Deflections (deg)	.351 .314 .438	.367 .350 .471
Control Rates (deg/sec)	41.9 36.9 51.6	42.8 37.0 52.1
Bending Moment (in lbs)	26,344	26,396
Shear (lbs)	492.0	492.9
Torque (in lbs)	1657.	1708.

Table 4.1 RMS Performance at Design Point for LQR and LQR/LTR Compensators.

	Eigenvalue	Zero	I	II
1	-13.8+430j	-6.017+346.9j	-6.017+346.9j	-6.017+346.9j
2	–	–	–	-
3	-19.2+275j	.1075	-.12	-4.8925
4	–	-.1075	-.1	-5.1075
5	-1.4+194j	-1.040+198.2j	-1.040+198.2j	-1.040+198.2j
6	–	–	–	-
7	-43.6+136j	.03479	-.04	-4.9652
8	–	-.03479	-.02	-5.0348
9	5.2+119.7j	.000003+.17j	-.000003+.17j	-5.000003+.17j
10	–	–	–	-
11	-92	-111.2	-111.2	-111.2
12	-113.6	-129.8	-129.8	-129.8
13	-112	-122.2+.06059j	-122.2+.06059j	-122.2+.06059j
14	-112.4	-122.2-.06050j	-122.2-.06059j	-122.2-.06059j
15	-112.2	-111.7	-111.7	-111.7
16	-179.9	∞	-879.9	-879.9
17	-125.6+287.9j	∞	-825.6+287.9j	-825.6+287.9j
18	–	∞	–	-
19	-175	∞	-875	-875
20	-127.4+291.9j	∞	-827.4+291.9j	-827.4+291.9j
21	–	∞	–	-
22	-185	∞	-885	-885
23	-123.9+283.9j	∞	-823.9+283.9j	-823.9+283.9j
24	–	∞	–	-
25	-.492	-.492	-.492	-492
26	-.497	-.497	-.497	-497

Table 4.2 Eigenvalue-Zero Assignments for Observer Design.

... the ... of ... group ... left ...

... of ... ... and assigned ... ...

... assigned ... negative real part and ... rest ... ...

... ... Note that in all cases the assigned eigenvalues have ... ... of the observer. The resulting gain matrix $H(q_o)$ is

... of $N(q_o)$ ...

$$ ... K(q_o) ... q_o ... $$

$$ ... H(q_o) x_j ... C(q_o) z_j $$

$$ \dot{z}(t) = A ... z + B(q_o)u + H(q_o)(y - C(q_o)z) \quad u = -K(q_o)z. $$

... of the system with compensator is indicated by the rms values listed in Table 4.1. ... of the maximum and minimum singular values of the return difference ... ... results appear in Figure 4.1.

Although the system with compensator appears to have good stability and performance properties the synthesized control laws are sensitive to changes in dynamic pressure. The locus of eigenvalue locations of $A(q) - B(q) K(q_o)$ with variation in dynamic pressure is depicted in Figure ... that at the evaluation points $q = 5.141, 5.537, 6.639$ lb/in^2 the system matrices $A(q) - B(q)K(q_o)$ are unstable. Additional calculations show that the matrices $A(q) - H(q_o)C$ possess
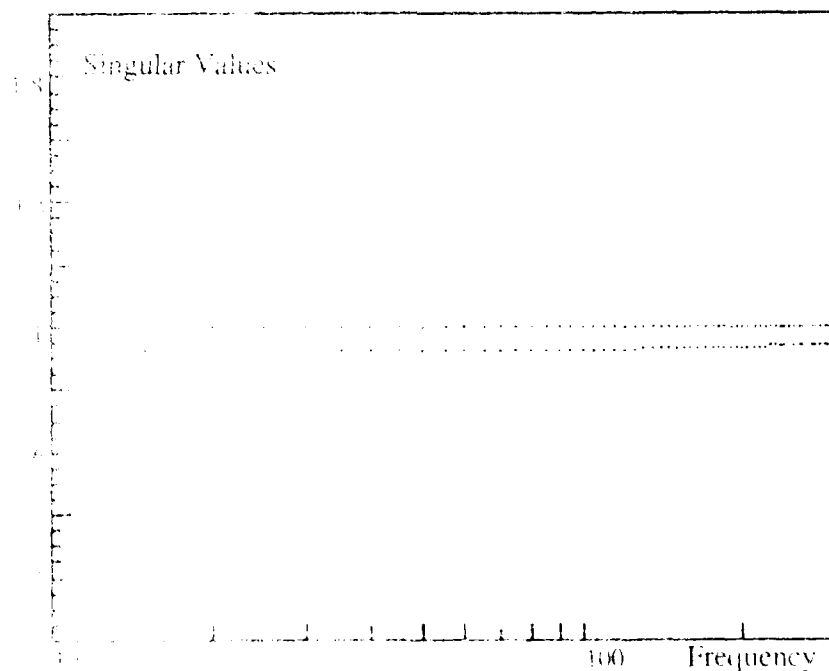

Figure ... Maximum and Minimum Singular Values of the Return
... Matrix ... $[A - BF - HG]^{-1}HG$ ... $[A - B]$
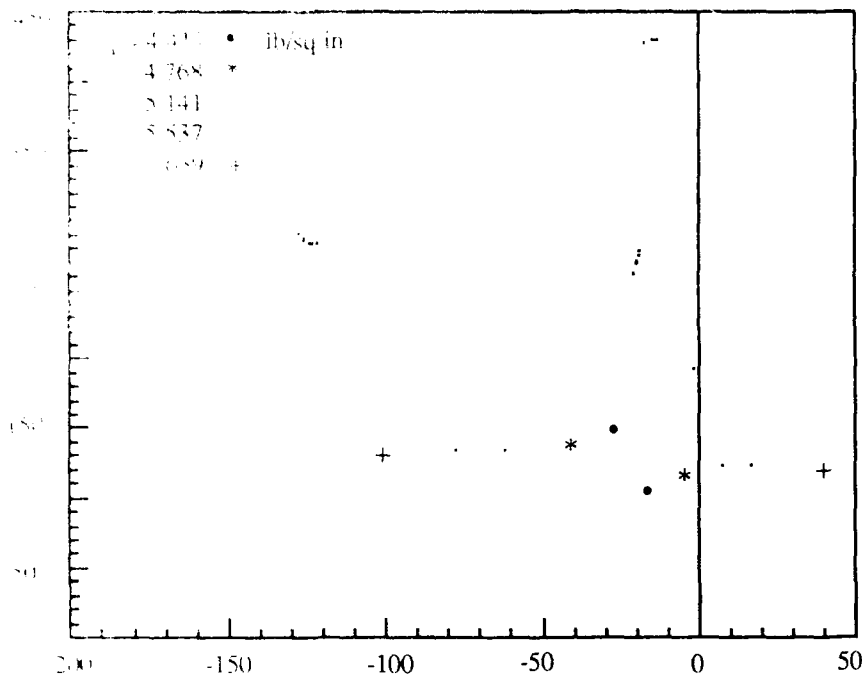
Figure 4.2 Locus of Pole Locations for LQR Compensators $A(q) - B(q)K(q_o)$.

unstable eigenvalues for q= 5.141 and 6.639 lb/in^2. It is desired to reduce this sensitivity while maintaining performance requirements by use of the eigenspace assignment methods discussed in the previous section.

To illustrate the design procedure, a design involving only a small number of parameters is undertaken. The minimum energy stabilizing controller $K(q_o)$ modifies the eigenstructure of $A(q_o)$ in a simple manner, here the unstable eigenvalues are flipped symmetrically with respect to the imaginary axis while all other eigenvalue/eigenvector pairs remain unchanged. That is the stabilizing, minimum energy feedback control law only alters the mode shape associated with the open loop instability. In the design example, the closed loop eigenvalues are left in the stable location achieved by the minimum energy controller $K(q_o)$ and an attempt is made to reduce the sensitivity of this control law to changes in dynamic pressure by modifying the eigenvector corresponding to the unstable mode. Letting λ denote the desired eigenvalue associated with the unstable pole, the attainable eigenvectors v and modified gain matrix K_M must satisfy

$$[\lambda I - A, B] \begin{bmatrix} v \\ K_M v \end{bmatrix} = 0$$

In this example the dimension of Ker$[\lambda I-A, B]$ is three. Since this is a complex vector space there are really six free parameters which describe the attainable eigenvectors and modified gain matrix. Thus the objective is to use the flexibility of eigenvector assignment to design a modified full state

feedback control law K_M which reduces sensitivity to changes in dynamic pressure, maintains performance requirements and is robust with respect to model uncertainties. Although all of these design objectives will not be met, the example does illustrate that many of them can be achieved by simply modifying the mode shape associated with the unstable eigenvalues.

The sensitivity of the unstable eigenvalue to changes in dynamic pressure is calculated from the formula

$$d\lambda /dq = \{ u' (d(A(q) - B(q)K(q_o)) /dq) v \} / (u' v)$$

where u' and v denote left and right eigenvectors of $A(q) - B(q)K(q_o)$ corresponding to the unstable eigenvalue and d($A(q) - B(q)K(q_o)$) /dq, is $d\lambda(q_o)$ /dq is estimated by the difference quotient $\{A(q) - B(q)K(q_o) - (A(q_o) - B(q_o)K(q_o)\}/(q - q_o)$. The sensitivity is $d\lambda /dq = 43.88 + 18.67j$. The design procedure is to utilize the freedom in eigenvector assignment in order to minimize the magnitude of $d\lambda(q_o)$ /dq subject to the stability and performance constraints at this value of dynamic pressure. Robustness constraints are not explicitly taken into consideration.

Let $W = [W_1', W_2']'$ be a basis for the ker[$\lambda I - A, B$] where $\lambda = -5.2 + 119.7j$. The unstable open loop eigenvalue is $5.2 \pm 119.7j$. The attainable eigenvectors are linear combinations of the columns of W_1, i.e. $v = W_1 c$ for an arbitrary vector c. The design parameters $\alpha = c$ are the components of an attainable eigenvector with respect to the basis W_1, which is obtained using the SVD as described in Section 2. Here $s(\alpha) = d \lambda/ dq$. The components of the performance vector $p(\alpha)$ are a stability indicator, position and rate rms values for each controller, and wing bending moment, shear, and torque rms values. The stability indicator was taken taken as the maximum of the real part of the closed loop system eigenvalues. In our design example the prescribed lower bounds were LB = (-10,000,0,0,0,0,0,0,0,0,0) and the prescribed upper bounds were UB = (0,3.7,15,15,372,372,372,30000,1000,2000). The weight matrices were $Q_1 = 1$ and $Q_2 = diag(10000,50,50,50,25,50,50,50,50,50)$.

A search over the six dimensional parameter space determines a desired eigenvector to achieve this minimum and the corresponding gain matrix K_M. The sensitivity achieved at this point is $d\lambda(q_o)$ /dq $= 10.16 + 2.07j$. The corresponding gain matrix is listed in Table A2 of Appendix A. To further illustrate the sensitivity reduction of this design the closed loop poles for the systems $A(q) - B(q)K_M$ for various values of dynamic pressure have been depicted in Figure 4.3. Note that stability is achieved at each evaluation point. Corresponding rms values are listed in Table 4.3. It is most intersesting that these results can be achieved by simply modifying the mode shape corresponding to the unstable mode. The maximum and minimum singular values of the return difference matrix $I + K_M C(q_o) (sI - A(q_o))^{-1} B(q_o)$ over a specified frequency range have been graphed in Figure 4.4. This figure indicates that the robustness properties of the minimum energy controller have not been maintained and that a more reasonable design would include singular value constraints.

Figure 4.3 Locus of Pole Locations for Modified LQR Compensators $A(q) - B(q)K_M$

Figure 4.4. Maximum and Minimum Singular Values of the Return
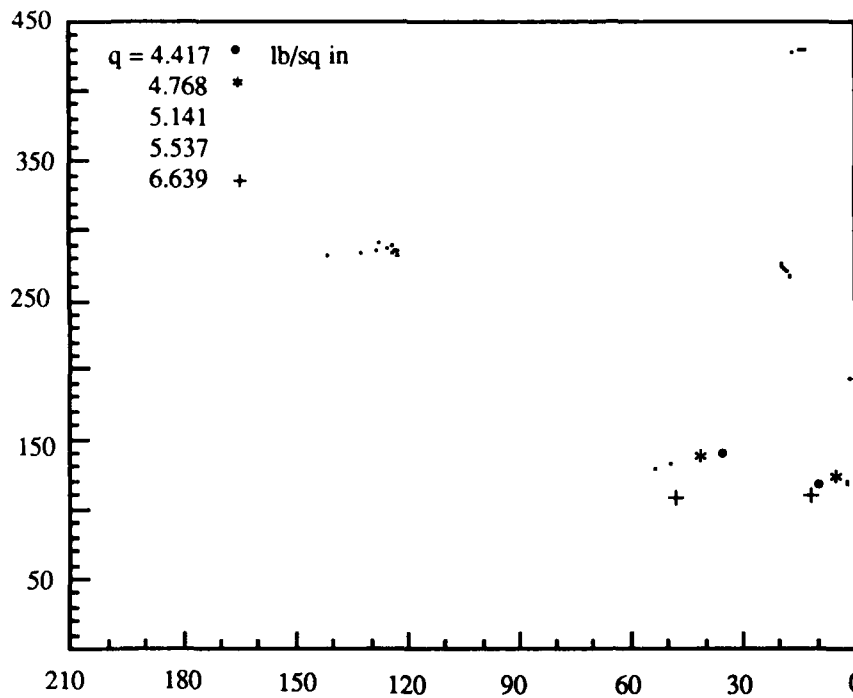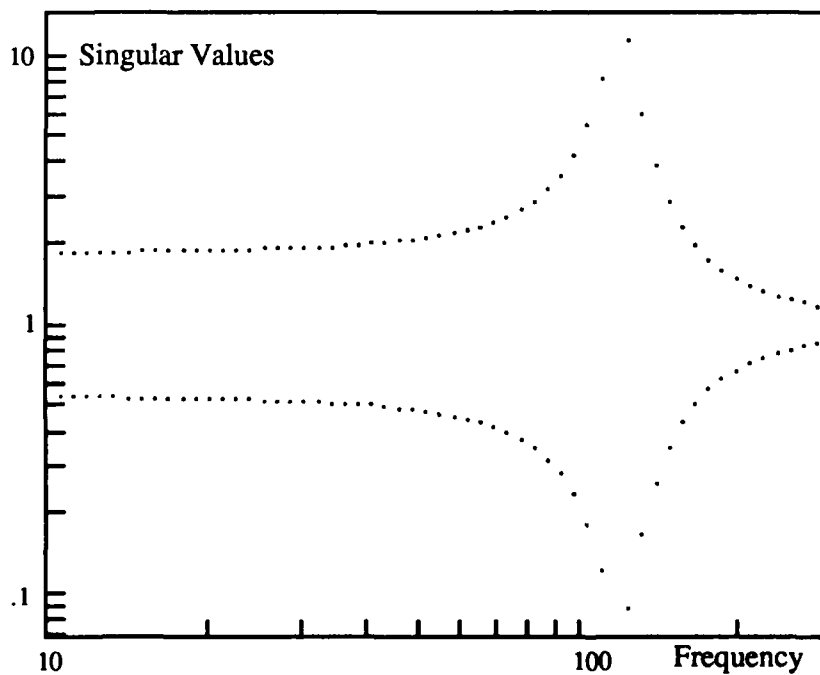Difference Matrix $I + K_M(sI - A)^{-1}B$.

Although the separation principle guarantees the stability of the compensator with feedback gain matrix $K = K_M$ and observer gain matrix $H = H(q_o)$, instabilities still arise at off design points due to use of the observer gain matrix $H = H(q_o)$. Consequently a modified observer design was attempted, the desired pole locations for this modified design are listed in column II of Table 4.2 and the resulting gain matrix H_M is listed in Table A4 of Appendix A . The overall performance of the compensator may now be evaluated. First, it should be noted that numerical computations verify the stability of the compensator at all evaluation points. Thus the design procedure, i.e. modification of the eigenvectors associated with the unstable mode at the design point for feedback gain matrix design and assignment of observer eigenstructure to corresponding system zeros and invariant directions for observer gain matrix design has resulted in a compensator whose stability is less sensitive to changes in dynamic pressure. However one should also note that the rms values for the system with compensator, see Table 4.4, indicate a high level of control activity at the evaluation points furthest from the design point. Further improvement in performance should be attainable though modification of other mode shapes and by adjusting the weight matrices in the design procedure to obtain an acceptable blend between sensitivity reduction and control activity. Plots of the maximum and minimum singular values of the return difference matrix $I + (sI - A + BK_M + H_MC)^{-1}H_MC(sI - A)^{-1}B$ are depicted in Figure 4.5. Again, robustness properties of the minimum energy controller have not been retained.

RMS \ Dynamic Pressure	4.417(lb/in^2)	4.4768	5.141	5.537	6.639
Control Deflections (deg)	3.008 .380 2.431	3.716 .4394 3.026	4.989 .5725 4.061	5.587 .6444 4.518	6.506 .9127 4.850
Control Rates (deg/sec)	376.0 42.66 307.4	465.4 52.0 380.6	609.8 67.10 498.4	659.9 71.38 538.6	350.8 35.83 285.9
Bending Moment (in lb) Shear (lbs) Torque (in lbs)	24,825 453.8 458.2	22,250 484.3 616.6	27,950 521.9 848.5	29,910 563.7 970.4	39,008 745.3 2143

Table 4.3 RMS Performance for Modified LQR Compensator $dx/dt = (A(q)-B(q)K_M)x$.

Dynamic Pressure					
RMS	4.417(lb/in²)	4.4768	5.141	5.537	6.639
Control Deflections (deg)	5.676	6.358	8.500	10.650	17.681*
	.3885	.5404	.7918	1.023	1.834
	4.169	5.185	6.895	8.587	13.739
Control Rates (deg/sec)	329.9	471.2	720.8	905.4*	644.5
	37.4	52.8	79.63	98.5	70.13
	272.6	387.2	589.0	738.9	516.4
Bending Moment (in lb)	25,322	26,678	28,436	30,553	40,719
Shear (lbs)	469.2	498.5	537.1	582.9	787.9
Torque (in lbs)	1093.8	1412.6	1888.1	2443.0	5831.0

* saturation

Table 4.3 RMS Performance for Modified LQR/LTR Compensator

$$dx/dt = A(q)x - B(q)K_M z + \Gamma(q)\eta, \quad dz/dt = H_M C(q)x + (A(q) - B(q)K_M - H_M C(q))z.$$
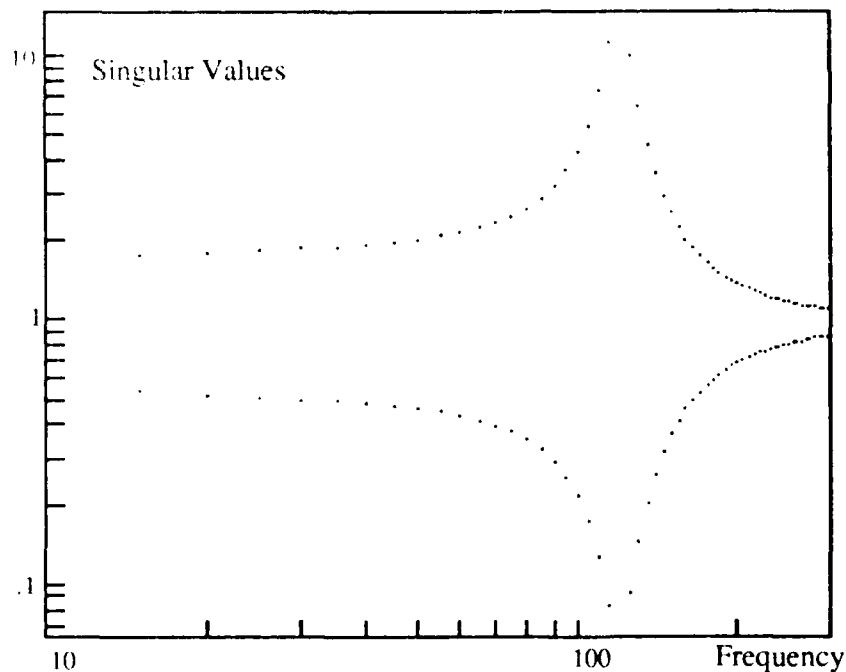

Figure 4.5. Maximum and Minimum Singular Values of the Return Difference Matrix $I + K_M (sI - A + BK_M + H_M C)^{-1} H_M C(sI-A)^{-1}B$.

CONCLUDING REMARKS.

A procedure based upon eigenstructure assignment to reduce control system sensitivity to changes in system parameters has been introduced. Through an extensive design example it has been shown that modification of a single mode shape can lead to a gain matrix whose sensitivity to parameter variation is significantly reduced. The design example also indicates that a proper blend must be achieved between sensitivity reduction and performance constraints. Another problem that needs to be addressed is the explicit incorporation of robustness requirements into the design procedure. Current effort involves the incorporation of explicit robustness constraints in the design. Additional freedoms in eigenvector assignment are being used to achieve this objective.

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Appendix A

Table A1. Gain matrix $K(q_0)$

COLUMNS 1 THROUGH 10

-6.8777D-02	-2.3731D-02	-1.2176D-02	-1.2614D-01	7.4400D-02	2.5776D-04	-8.2104D-04	-6.2575D-04	1.0233D-03	2.8986D-04
-7.6158D-02	1.2296D-02	2.5868D-02	-5.9012D-02	5.0607D-02	-8.5410D-05	-5.2110D-04	-4.1985D-04	-6.0049D-04	1.7102D-04
-1.0114D-01	6.0809D-03	2.3898D-02	-9.9802D-02	7.5650D-02	-1.4896D-05	-7.9513D-04	-6.3011D-04	-9.3896D-04	2.6696D-04

COLUMNS 11 THROUGH 20

2.2653D-05	-1.7254D-05	-2.4099D-05	-3.3100D-05	9.3842D-06	7.2840D-02	1.5181D-04	3.6651D-07	7.5082D-02	1.3197D-04
1.3911D-05	-1.6472D-05	-2.2146D-05	-2.9093D-05	8.2629D-06	5.0690D-06	1.0890D-04	2.9298D-07	5.9532D-02	1.1558D-04
2.1443D-05	-2.2574D-05	-3.0616D-05	-4.0644D-05	1.1539D-05	7.5278D-02	1.6035D-02	4.1907D-07	8.5332D-02	1.6161D-04

COLUMNS 21 THROUGH 26

2.9286D-07	6.1518D-0	2.0386D-04	4.1890D-07	7.1832D-05	2.5127D-04
2.7783D-07	4.8599D-02	1.6546D-04	3.8131D-07	1.6355D-06	6.3676D-06
3.8131D-07	6.9727D-02	2.3574D-04	5.2831D-07	2.2871D-05	8.0690D-05

Table A2. Modified Gain Matrix K_M

COLUMNS 1 THRU 10

6.6737D-01	-9.8587D-01	-1.1225D+00	-1.3192D+00	2.7994D-01	9.1902D-03	-2.5360D-03	-4.2674D-03	6.8621D-03	1.9148D-03
-1.2505D-01	5.5041D-02	7.8027D-02	-2.4022D-02	5.4389D-02	-4.7527D-04	-4.4275D-04	-5.0509D-04	7.0485D-04	1.4536D-04
-4.6255D-01	8.0337D-01	9.0046D-01	1.1654D+00	-2.9294D-01	-7.5240D-03	2.6075D-03	4.1656D-03	6.4136D-03	-1.7936D-03

COLUMNS 11 THRU 20

1.3245D-04	8.4395D-05	9.0749D-05	8.2844D-05	-2.3953D-05	2.3801D-01	3.9338D-04	1.8722D-05	1.8730D-02	3.1730D-04
1.2748D-05	-2.4668D-05	-3.2265D-05	-4.0941D-05	1.1645D-05	5.6160D-02	1.2533D-04	3.7413D-05	7.6426D-02	3.6219D-04
-1.2658D-04	-5.0297D-05	-4.8771D-05	-3.3888D-05	9.9770D-06	-2.5824D-01	-4.5692D-04	3.5259D-05	5.4382D-02	1.2463D-04

COLUMNS 21 THRU 26

-1.3768D-06	1.8516D-02	-7.7638D-05	-1.4621D-06	1.7591D-03	6.1331D-03
4.1423D-07	6.2165D-02	2.1723D-04	5.5154D-07	-6.7698D-05	-2.3516D-04
8.0965D-07	-7.3594D-02	-1.3382D-04	7.5640D-07	-1.4617D-03	-5.0971D-03

```
COLUMNS  1 THRU 10
 2.1421D-04   1.3433D-04   2.3752D-05   7.5583D-05   1.0461D-04  -4.2480D-01  -3.8760D-01  -1.4059D-01   4.2158D-02  -1.8249D 01
-4.8413D-05  -2.3273D-05   3.2657D-05   1.8088D-05   3.1120D-05  -1.1226D-02  -6.9789D-02  -5.3708D-02  -1.7591D-01   3.8875D 02
-1.8057D-04  -1.4981D-04  -5.3688D-05  -5.9963D-05  -7.9142D-05   5.3004D-01   3.7156D-01   1.5721D-01   8.6324D-02   3.9778D-02

COLUMNS  11 THRU 20
 2.9907D-01   9.6723D-01  -8.7533D-02  -2.5974D+00   6.7378D-01  -2.1786D-01  -1.1175D+02   1.6856D+04   1.7126D-01   9.0704D+01
-3.0534D-01  -3.2641D+00  -1.3896D+00  -1.5610D+00   2.4344D+00  -2.5633D-02  -3.6712D+00   2.9030D+-03   5.1764D-03   5.4177D+00
-8.0788D+00   1.2606D+01   7.0306D+00   1.0851D+01  -1.0952D+01   2.4771D-01   1.2130D+02  -2.0372D+04  -1.4001D 01  -7.8104D+01

COLUMNS  21 THRU 26
-1.2202D+04   8.5319D-03   3.1239D+01  -1.8131D+02  -1.7301D-06   8.7255D-07
-6.2941D+01   6.5042D-02   4.7196D+01  -5.4238D+03   5.5472D-06  -2.7974D-06
 9.3222D+03  -2.7438D-02  -6.6348D+01  -1.7363D+02   5.3353D-07  -2.6919D-07
```

Table A3. Transpose of Observer Gain Matrix $H(q_0)$

```
COLUMNS  1 THRU 10
 5.1686D-03   6.3101D-03   2.5002D-03   8.5485D-04   1.9471D-04  -4.1136D-01  -3.7167D-01  -1.3447D-01   4.4787D-02  -1.8129D-01
-9.6559D-03   1.0550D-03   6.4969D-04   7.1452D-04  -7.6342D-05  -3.5858D-02  -6.7233D-02  -5.1815D-02  -1.7400D-01   3.8932D-02
-3.2816D-03  -7.0551D-03  -3.1050D-03  -1.6631D-03   1.3812D-04   5.2171D-01   3.5376D-01   1.4939D-01   8.1785D-02   3.9618D-02

COLUMNS  11 THRU 20
 2.1344D-01   7.9300D-01  -1.0288D-01  -2.3525D+00   6.5868D-01  -1.8375D-01  -1.1380D+02   1.5759D+04   1.4318D-01   9.2305D+01
-1.3536D 01  -3.2364D+00  -1.4035D+00  -1.6124D+00   2.4275D+00  -2.4222D-02  -3.9269D+00   2.8630D+03   3.3690D-03   5.4731D+00
-8.1568D+00   1.2767D+00   7.0651D+00   1.0660D+01  -1.0940D+01   2.1094D-01   1.2364D+02  -1.9181D+04  -1.1595D-01  -7.9400D+01

COLUMNS  21 THRU 26
-1.1315D+04   5.3386D-04   3.1304D+01   1.2741D+02  -2.3119D-04   1.1669D-04
-4.4181D+00   5.1688D-02   4.7809D+01  -4.9547D+03   5.1630D-04  -2.6063D-04
 8.5639D+03  -9.3615D-03  -6.6574D+01  -8.3205D+02   1.2552D-04  -6.3353D-05
```

Table A4. Transpose of Modified Observer Gain H_M

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Report Documentation Page

1. Report No. NASA CR-181618 ICASE Report No. 88-9	2. Government Accession No. *AD-A192 758*	3. Recipient's Catalog No.
4. Title and Subtitle AN APPLICATION OF EIGENSPACE METHODS TO SYMMETRIC FLUTTER SUPPRESSION		5. Report Date January 1988
		6. Performing Organization Code
7. Author(s) Robert E. Fennell		8. Performing Organization Report No. 88-9
		10. Work Unit No. 505-90-21-01
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No. NAS1-18107
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225		
		14. Sponsoring Agency Code

15. Supplementary Notes
Langley Technical Monitor: Submitted to Proc. AIAA Guidance, Richard W. Barnwell   Navigation, and Control Conference Final Report

16. Abstract

An eigenspace assignment approach to the design of parameter insensitive control laws for linear multivariable systems is presented. The control design scheme utilizes flexibility in eigenvector assignments to reduce control system sensitivity to changes in system parameters. The methods involve use of the singular value decomposition to provide an exact description of allowable eigenvectors in terms of a minimum number of design parameters. In a design example, the methods are applied to the problem of symmetric flutter suppression in an aeroelastic vehicle. In this example the flutter mode is sensitive to changes in dynamic pressure and eigenspace methods are used to enhance the performance of a stabilizing minimum energy/linear quadratic regulator controller and associated observer. Results indicate that the methods provide feedback control laws that make stability of the nominal closed loop systems insensitive to changes in dynamic pressure.

17. Key Words (Suggested by Author(s)) stabilization, eigenspace methods, flutter suppression	18. Distribution Statement 08 - Aircraft Stability and Control Unclassified - unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 21	22. Price A02

NASA FORM 1626 OCT 86

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